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Electroweak double-logs at small x

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Abstract: We investigate enhanced EW corrections to inclusive hard processes in the TeV energy region with emphasis on the small-x situation, in which the hard scale Q is significantly smaller than the available energy $\sqrt{s} \equiv Q/x$. We first propose and justify a general factorization formula in which the (double-log) EW form factor at scale Q^2 is factorized from EW parton distribution functions, which satisfy evolution equations of DGLAP type. We then investigate the small-x behavior of the EW parton distributions including the novel ones for non-vanishing t-channel weak isospin T and we compare it with a BFKL-type approach. In either approach we find that large small-x corrections of order $\alpha_w \log x \log Q^2/M^2$ (M being the EW symmetry breaking scale) are present only for $T = 2$ and not for $T = 1$, thus affecting only transverse WW interactions. We finally note that the $T = 2$ component is turned off in flavour-blind measurements at LHC, which therefore feel just the EW form factor at scale Q^2.

Keywords: Standard Model, Parton Model, Hadronic Colliders.

Contents

1. Introduction

It is by now clear that electroweak radiative corrections at the TeV scale [1, 2] have a much richer structure and a higher phenomenological relevance than one could have thought, say, ten years ago. The size of the corrections, for one thing, is typically of the order of 20-30 %, much bigger than the LEP *permille* level. This is due to the fact that the EW (Electro-Weak) corrections grow like the log square of the c.m. energy, which in turn is tied to the infrared structure of the theory [2] and opens up the possibility of resumming leading effects, with techniques partly mutated from QCD [3].

Even more interestingly, the infrared structure of the electroweak sector is radically different from QED or QCD, due to spontaneous symmetry breaking. As a result, the double log dependence on the infrared cutoff, which is physical and of the order of the weak scale, is present in both exclusive *and* fully (or partly) inclusive observables,* a phenomenon baptized "Bloch-Nordsieck violation" [4, 5]. This is a very interesting and striking fact from a theoretical point of view; phenomenologically, it means that considering the possibility of weak W,Z gauge bosons emission is more important than one could think. In fact, in the case of ILC physics EW corrections can dominate over the QCD ones [4], and even for an environment which is *a priori* dominated by QCD, like the LHC, studying weak boson emission turns out to be important [6, 7].

*The "inclusive" qualification refers, as usual, to observables which may restrict the nature of the "hard" final state (identified particles or jets, carrying some hard scale Q) but sum otherwise over all unobserved particles, including final state unobserved weak gauge bosons.

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In the above framework, various kinds of electroweak corrections for TeV-scale observables have been considered: exclusive observables — with an extensive literature on one loop results [8] and two loop calculations [9] — and inclusive observables [10], featuring the noncancellation phenomenon. In the latter case, the hard subprocess scale Q^2 has always been assumed to be of the same order as the initial c.m. energy s. This is the case for $e^+e^- \to jjX$ where j represents a final jet and X includes gauge bosons radiation, when $s \gtrsim Q^2 \gg \mu^2$, Q^2 being the 2 final jets invariant mass and μ the infrared cutoff scale, of the order of the weak scale.

The purpose of this work is to investigate the behavior of high energy electroweak corrections to inclusive observables when the hard subprocess scale Q^2 is significantly different from the c.m. initial scale s, i.e. when $\frac{Q^2}{s}$ becomes small.

The above problem is relevant and interesting under several aspects. Firstly, from a phenomenological point of view many processes relevant for both LHC and ILC physics (like Z and W production, Higgs production and so on) are characterized by relatively small values of x. Secondly, from a theoretical point of view the issue of factorization of electroweak corrections, because of the presence of uncanceled double logs and three different scales Q^2, s, μ^2 is far for being trivial. On top of this, the presence of big corrections related to the collider scale s of the form $\log \frac{s}{\mu^2}$ would mean that electroweak corrections receive huge enhancements even for small Q^2 values: addressing this issue is of course of paramount importance. To end with, as we shall see, small-x electroweak physics is qualitatively different from QCD; namely the relationship between DGLAP [11] and BKFL [12] approaches has a richer structure because of the uncanceled double-logs in inclusive observables.

2. Flavour structure of inclusive electroweak double-logs

Let us first recall the flavour structure[†] of inclusive electroweak corrections which are infrared sensitive, and thus contain, in the TeV energy region, double logarithms of the symmetry breaking scale. The effective coupling for such corrections is

$$\alpha_{\text{eff}}(Q^2) \equiv \alpha_W \log^2(Q/M_W) \quad (\alpha_w \equiv g_w^2/4\pi) \tag{2.1}$$

which is of the order $\alpha_{\text{eff}} = 0.2$ at the TeV threshold. The treatment of such corrections has been performed so far at both double-log and single-log accuracy, by proposing evolution equations [10] which generalize the DGLAP equations [11] to a non-trivial flavour structure. Our final purpose is to investigate such equations at small Bjorken-x, and to compare with the analogue BFKL evolution equation [13]. Before doing that, it is useful to understand the flavour dependence of such corrections in the eikonal limit – which takes into account double logs while neglecting single logs — and then to look at the collinear factorization structure for $Q^2/s \ll 1$.

We shall first consider (partly) inclusive observables which are flavour-blind (in the sense that final state flavour is not measured), and are triggered by initial fermions or

[†]By "flavour" here we mean "weak isospin or hypercharge quantum numbers". We only consider one family, and therefore neglect family mixing.

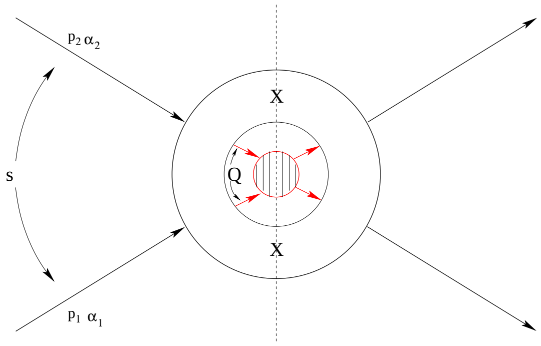

Figure 1: $2 \leftrightarrow 2$ overlap matrix for a Drell-Yan-type process, with two initial states of flavour $\alpha_{1,2}$ and momenta $p_{1,2}$ with $(p_1+p_2)^2 = s$. In the center (in red), the observed flavour-blind process at hard scale Q. Inside X we schematically include the undetected background (from QCD, QED and possibly IR and collinear W emissions).

bosons. Typical examples, for ILC physics would be just $e^+\ e^- \to$ hadronic jets or, for LHC physics, $q\ \bar{q} \to$ jets or $W^+\ W^- \to$ jets. In such cases, no flavour is registered in the final state, while the initial states carry their own, and we shall focus on their momenta and weak isospin, denoted by $(p_1, \alpha_1; p_2, \alpha_2)$ for the two incoming particles. These hard processes are therefore typically of Drell-Yan type. Due to the inclusive nature of the cross-sections so defined, final state singularities cancel out [4], and the remaining dependence is on initial state isospin indices, so that the flavour dependence of the squared amplitude can be arranged in an overlap matrix with two initial and two "final" indices, which actually double the initial ones.[‡] We shall comment later on the possible generalization to registered flavour in the final state, a case in which the overlap matrix involves more indices.

To be more precise, in order to describe the isospin dependence of EW corrections, one can define the overlap matrix illustrated in figure 1, which generalizes the cross-section for the flavour dependence, and is given in terms of the T-matrix as follows [4] (α_i, β_i are the initial state isospin indices):

$$\langle p_1\beta_1, p_2\beta_2|\ T^\dagger\ T\ |p_1\alpha_1, p_2\alpha_2\rangle = \ \mathcal{O}^{p_1p_2}_{\beta_1\alpha_1,\beta_2\alpha_2} \tag{2.2}$$

which can be roughly related by the optical theorem to the imaginary part of the forward amplitude. The observable cross sections are then obtained for diagonal flavour indices as follows:

$$d\sigma_{\alpha_1\alpha_2} = \mathcal{O}^{p_1p_2}_{\alpha_1\alpha_1,\alpha_2\alpha_2}\ d\Phi \tag{2.3}$$

where we define the overlap matrix to be dimensionless, so that the phase space is meant to be rescaled by a proper power of s. The SU(2) generators $t_i^a, t_i'^a, a = 1,2,3 \quad i = 1,2$ act on the overlap matrix as in the following example

$$(t_1^a\mathcal{O})_{\beta_1\alpha_1,\beta_2\alpha_2} = \sum_{\alpha_1'}(-t^a_{\alpha_1'\alpha_1})\mathcal{O}_{\beta_1\alpha_1',\beta_2\alpha_2} \qquad (t_1'^a\mathcal{O})_{\beta_1\alpha_1,\beta_2\alpha_2} = \sum_{\beta_1'} t^a_{\beta_1\beta_1'}\mathcal{O}_{\beta_1'\alpha_1,\beta_2\alpha_2} \tag{2.4}$$

[‡]Of course, in a measurement in a hadronic accelerator like LHC, the initial flavour states of quarks and Ws are weighted by the relevant luminosities, which have to be taken into account in order to compute the actual cross-section.

and are of course dependent on the representation of the considered i-th particle.

Virtual and real emission of soft gauge bosons in the initial state for the overlap function (2.2) in figure 1 is provided, at leading double-log level, by the external line insertion of the eikonal current

$$\boldsymbol{J}^{\mu}(k) = g_w \sum_{i=1}^{2} \boldsymbol{T}_i \frac{p_i^{\mu}}{p_i \cdot k} = \boldsymbol{T}_1 \left(\frac{p_1^{\mu}}{p_1 \cdot k} - \frac{p_2^{\mu}}{p_2 \cdot k} \right) \equiv \boldsymbol{T}_1 j_{12}^{\mu}(k) \tag{2.5}$$

where k is the momentum of the emitted soft gauge boson, p_i the i-th leg momentum, g_w the SU(2) gauge coupling and we have defined the total (t-channel) isospin generator referred to the leg i as $T_i \equiv t_i + t_i'$ [4]. Notice that the part of the current proportional to g' is absent altogether because of the cancellation of the abelian components for inclusive observables [4]. Note, furthermore, that we have used in (2.5) isospin conservation to set $\boldsymbol{T}_2 = -\boldsymbol{T}_1$. In fact, since we consider energy scales of the order of 1 TeV and beyond, we take all particles to be massless and we work in the limit in which the SU(2) $\otimes$ U(1) symmetry is recovered. In other words the overlap matrix is invariant under total isospin transformation:

$$T^a_{\mathrm{tot}} \equiv \sum_i T^a_i \qquad \exp[\alpha^a T^a_{\mathrm{tot}}]\mathcal{O} = \exp[\boldsymbol{\alpha} \cdot \boldsymbol{T}_{\mathrm{tot}}]\mathcal{O} = \mathcal{O} \quad \Rightarrow \quad \boldsymbol{T}_{\mathrm{tot}}\mathcal{O} = 0 \tag{2.6}$$

The emission probability of real and virtual bosons off the initial legs is then obtained by squaring the eikonal current so as to obtain the insertion operator

$$\frac{1}{2}\boldsymbol{J}_{\mu}\boldsymbol{J}^{\mu} = -g_w^2 \frac{p_1 p_2}{(kp_1)(kp_2)} \boldsymbol{T}_1^2; \qquad -\boldsymbol{T}_1^2 = -\mathbf{t}_1^2 - \mathbf{t'}_1^2 - 2\mathbf{t}_1 \cdot \mathbf{t'}_1 = -2C_1 - 2\mathbf{t}_1 \cdot \mathbf{t'}_1 \tag{2.7}$$

which in turn provides the eikonal radiation factor for gauge boson emission:

$$L_W(s) = \frac{g_w^2}{2} \int_M^E \frac{d^3\boldsymbol{k}}{2\omega_k (2\pi)^3} \frac{2p_1 p_2}{(kp_1)(kp_2)} = \frac{\alpha_w}{4\pi} \log^2 \frac{s}{M^2}, \quad \alpha_w = \frac{g_w^2}{4\pi} \tag{2.8}$$

where the k-integral has been performed over the region where the soft momentum fraction $1 - z \equiv (p_1 - k)_+/\sqrt{s}$ satisfies the bound

$$(1 - z)\sqrt{s} > |\mathbf{k}| > M_Z \simeq M_W \equiv M \tag{2.9}$$

in both the forward and backward hemispheres with respect to the incoming momentum p_1. Finally, by iterating the procedure over any number of soft bosons, we get the resummed expression for the overlap matrix as the simple exponential

$$\mathcal{O}(s) = \exp[L_W(s)\boldsymbol{T}_1 \cdot \boldsymbol{T}_2]\mathcal{O}^H = \exp[-L_W(s)(\boldsymbol{T}_1^2 + \boldsymbol{T}_2^2)/2] = \exp\left[-L_W(s)\boldsymbol{T}_1^2\right] \mathcal{O}^H \tag{2.10}$$

Note that, due to the simple relation in eq. (2.7) of the total t-channel isospin to the Casimir operators of the colliding particles, real emission ($\sim -2\mathbf{t}_1 \cdot \mathbf{t'}_1$) occurs with weight $(2C_1 - \boldsymbol{T}^2)$, relative to the virtual corrections provided by $2C_1$. This structure will be essentially kept at single-log level as well, in the DGLAP and BFKL approaches that we shall consider next for $Q^2/s \ll 1$.

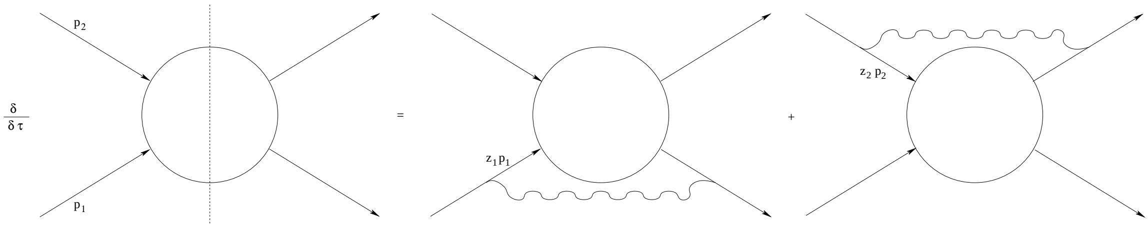

Figure 2: General structure of the Collinear Evolution Equations for the $2 \leftrightarrow 2$ overlap matrix. The wavy lines denote real boson emission with momentum rescaling.

3. Collinear factorization and EW evolution equations

Let us now recall the collinear evolution equations for the overlap matrix, with the purpose of resumming single and double EW logarithms. Such equations were derived in [10], where, with the help of collinear Ward Identities and working in the Feynman gauge, it was shown that the only relevant diagrams are those illustrated in figure 2.

Here, in order to discuss the small-x limit, we specialize to the case of initial bosons and we shall simplify it by omitting the mixing with the fermionic channels. Then the above procedure results in the following bosonic channel evolution equation for the overlap matrix at given initial energy-squared s, measurement hard scale Q^2 and t-channel isospin $\boldsymbol{T}^2 = T(T+1)$:

$$\begin{aligned} \frac{\partial \mathcal{O}^{(T)}(s,Q^2,\mu^2)}{\partial \tau} &= \frac{\alpha_W}{\pi} \left\{ 2\,p^V\, \frac{\mathbf{T}^2}{2}\, \mathcal{O}^{(T)}(s,Q^2,\mu^2) \right. \\ &\quad + \left(C_A - \frac{\mathbf{T}^2}{2} \right) \int \frac{dz_1}{z_1} P(z_1)\, \mathcal{O}^{(T)}(sz_1,Q^2,\mu^2) + \\ &\quad \left. + \left(C_A - \frac{\mathbf{T}^2}{2} \right) \int \frac{dz_2}{z_2} P(z_2)\, \mathcal{O}^{(T)}(sz_2,Q^2,\mu^2) \right\} \end{aligned} \tag{3.1}$$

$$\begin{aligned} P^V &= p^V \delta(1-z), \quad p^V = -\left(\frac{1}{2}\log\frac{s}{\mu^2} - \frac{11}{12} \right) \\ P^R &= \left(z(1-z) + \frac{z}{1-z} + \frac{1-z}{z} \right) \theta\left(1 - z - \frac{\mu}{\sqrt{s}} \right); \quad P(z) = \lim_{\frac{\mu}{\sqrt{s}} \to 0} (P^V + P^R) \end{aligned} \tag{3.2}$$

where $\tau \equiv \log(Q^2/\mu^2)$ is the evolution parameter, $\mu^2 < \mathbf{k}^2$ is an infrared cutoff on transverse momenta (to be set equal to the symmetry breaking scale at the end) and z_1 (z_2) is the momentum fraction variable for DGLAP splitting on leg 1 (2).

Note that, as emphasized in [10], the EW evolution equation (3.1) is to be seen as a "soft" evolution equation in the cutoff parameter μ^2 on the initial legs' transverse momenta, and that — due to the presence of the uncanceled double-logs — it is not obviously equivalent to an "ultraviolet" evolution equation in Q^2. For this reason the P^V distribution is dependent on the cutoff parameter μ^2/s, where s, by (2.9), is the relevant variable for eikonal insertion in the initial state. Furthermore, the real emission density P^R incorporates the soft emission cutoff in (2.9), while $P(z)$ is the regularized one, obtained by combining P^R and P^V. The normalization of $P(z)$ differs by a factor of 2 from the customary one, so that $P(z) \sim 1/z$ for $z \to 0$.

The overlap evolution equation (3.1) was already used in [10] to introduce bosonic (or fermionic) PDFs and to derive DGLAP-type equations for them in the case in which s and Q^2 are of the same order. Here we want to generalize this procedure to the small-x region, where the collinear factorization has to specify which scale, Q^2 or s, has to carry the EW double-logs in order to be able to factorize the appropriate PDFs. We shall show that Q^2 is the appropriate choice and, to this purpose, we propose the following factorization ansatz:

$$\mathcal{O}^{(T)}(s, Q^2, \mu^2) = \int \frac{dx_1}{x_1}\frac{dx_2}{x_2} \, \exp\left[-\frac{\alpha_w}{2\pi}\frac{\mathbf{T}^2}{2}\left(\log\frac{Q^2}{\mu^2}\right)^2\right] \, f^{(T)}(x_1, Q^2, \mu^2) \times \mathcal{O}_H^{(T)} \delta\left(\frac{Q^2}{x_1 x_2 s} - 1\right) \, f^{(T)}(x_2, Q^2, \mu^2) \tag{3.3}$$

where μ^2 is the off shell value of the minimal $k_\perp^2$ and should be set equal to M^2 for the physical overlap function. Here the bosonic PDFs $f_i^{(T)} \equiv f^{(T)}(x_i, Q^2, \mu^2))$ (normalized so that $x f_i^{(T)}$ are quasi-constant in the small-x region) are supposed to be free of double-logs in Q^2/M^2 but still contain $\log Q^2/\mu^2$ and $\log 1/x$ enhancements, while $\mathcal{O}_H$ should not contain any collinear or high-energy logarithms. The above factorization property is a non-trivial extension of the one valid in QCD, because — for $\mathbf{T}^2 \neq 0$ — it is meant to control both EW double-logs and single collinear logs.

In order to derive the factorization ansatz (3.3) at collinear level, we replace it in (3.1) by neglecting for simplicity the single-logs generated by p^V (they can be restored later on) because we are mostly interested in controlling double-logs of IR-collinear type and mixed ones, involving high-energy logarithms $\log 1/x$. We then obtain an evolution equation involving the product of PDFs:

$$\frac{\partial}{\partial \tau}\left[f_1^{(T)} f_2^{(T)}\right] = -\frac{\alpha_w}{\pi}\frac{\mathbf{T}^2}{2}\left(\log\frac{1}{x_1} + \log\frac{1}{x_2}\right) f_1^{(T)} f_2^{(T)} + \frac{\alpha_w}{\pi}\left(C_A - \frac{\mathbf{T}^2}{2}\right) \times \left[(P \otimes f_1^{(T)}) f_2^{(T)} + f_1^{(T)} (P \otimes f_2^{(T)})\right] \tag{3.4}$$

where $P \otimes f(x) \equiv \int \frac{dz}{z} P(z) f(\frac{x}{z})$, and we note that the relation $s = Q^2/x_1 x_2$ has generated the additive virtual term $\sim \log(x_1 x_2)$. Finally, by dividing by $f_1 f_2$ we obtain the single leg evolution equation:

$$\frac{\partial f^{(T)}(x,\tau)}{\partial \tau} = -\frac{\alpha_w}{\pi}\frac{\mathbf{T}^2}{2}\log\frac{1}{x}\, f^{(T)}(x,\tau) + \frac{\alpha_w}{\pi}\left(C_A - \frac{\mathbf{T}^2}{2}\right) P \otimes f^{(T)}(x,\tau), \left(0 \leq \tau \leq t \equiv \log\frac{Q^2}{M^2}\right) \tag{3.5}$$

which therefore exhibits the desired factorization. We remark the role of choosing the Q^2 scale for the double-log factor in (3.3), in the above derivation of factorized evolution equations. Had we chosen the energy variable $s = Q^2/x_1 x_2$, it would have generated nonfactorizable double logs of type $\log x_1 \log x_2$ thus violating the factorized structure of (3.4).

We can also investigate inclusive hard processes in which some flavour (weak isospin) state is identified in the final state for one or more particles, similarly to multiparticle

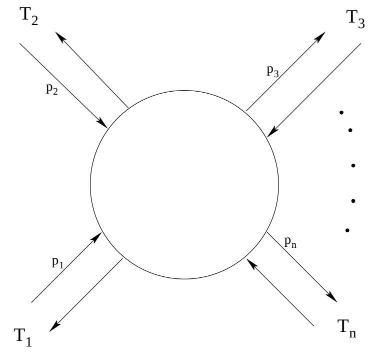

Figure 3: Overlap matrix with n registered flavour isospin legs (they can be both in the initial that in the final state)

distributions in QCD. The corresponding overlap function is now an n by n matrix, as depicted in figure 3, where we denote initial t-channel isospins by $\boldsymbol{T}_1$ and $\boldsymbol{T}_2$ and final ones by $\boldsymbol{T}_3, \ldots, \boldsymbol{T}_n$. We believe that a factorization formula exists in this case also, by singling out an n-particle form-factor computed by the eikonal current insertion as in the $2 \to 2$ case, while the collinear logarithms are factored out in PDFs or final state fragmentation functions. The situation is here similar to that occurring in QCD when the phase-space boundary or some veto expose a double-log behaviour in infrared sensitive observables [14]. As such, this problem has been investigated since quite a time [15] and, most recently, in [16]. The eikonal squared current is well known for $n = 3$. By using isospin conservation $(\boldsymbol{T}_1 + \boldsymbol{T}_2 + \boldsymbol{T}_3 = 0)$ we can generalize eqs. (2.5) and (2.7) to obtain

$$\frac{1}{2}\boldsymbol{J}_\mu \boldsymbol{J}^\mu = \frac{1}{2}\, \boldsymbol{T}_1^2\, j_{12}(k) \cdot j_{13}(k) \;+\; \text{cyclic} \qquad (n = 3) \tag{3.6}$$

where the product of currents in front of $\boldsymbol{T}_i^2$, by the definition in (2.5), is collinear singular only for $\mathbf{k}$ in the direction of p_i. Therefore, at double-log level, the eikonal radiation exponent is simply additive in the isospin charges, as follows

$$\mathcal{O}_n^{(T)} \sim \exp(\mathcal{L}_W^{12\cdots n}), \quad \mathcal{L}_W^{12\cdots n} = -\frac{1}{2}\, L_W(Q^2) \sum_{i=1}^{n} \boldsymbol{T}_i^2 + \text{single logs} \tag{3.7}$$

A similar result holds for $n = 4$, except that the single IR logarithms should now be computed by the techniques described in [15, 16].

The novel feature of the evolution equation (3.5) for the PDFs– compared to the DGLAP equations in QCD — is the $\log 1/x$ behaviour of the diagonal term for $\boldsymbol{T}^2 \neq 0$, which suggests the existence of extra *damping* due to mixed infrared and high-energy logarithms ($\tau\ log\frac{1}{x}$), to be looked at in detail. In fact, in this nontrivial-flavour evolution, such suppression could modify in an important way the basic form-factor behaviour already factored out in (3.3) (the factor $\exp[-\frac{\alpha_w}{2\pi}\frac{\mathbf{T}^2}{2}(\log\frac{Q^2}{\mu^2})^2]$). We shall basically investigate that in the following, by using both DGLAP and BFKL approaches. We further notice that the flavour factors occurring in (3.5) for the form-factor vs. real-emission terms are indeed the same as in the eikonal treatment recalled before, where the relevant Casimir is now that of the adjoint representation, $C_A = 2$. The possible values of $\boldsymbol{T}^2 = T(T+1)$ are instead provided by $T = 0, 1, 2$ as usual.

4. Bosonic DGLAP-type equation in the small-x region

Here we shall solve by customary methods the bosonic equation (3.5) for the various T values, by focusing on its small-x behaviour. By introducing the Mellin transform variable $\omega \equiv N-1$, where N is the customary moment index

$$\tilde{f}(\omega) = \int_0^1 dx\, x^{\omega}\, f(x)\,, \qquad x f(x) = \frac{1}{2\pi i}\int_{c-i\infty}^{c+i\infty} d\omega\, \tilde{f}(\omega)\, x^{-\omega} \tag{4.1}$$

the basic equation (3.5) can be rewritten in differential form

$$\frac{\partial \tilde{f}^{(T)}(\omega,\tau)}{\partial \tau} = \frac{\alpha_w}{\pi}\frac{\mathbf{T}^2}{2}\frac{\partial \tilde{f}^{(T)}(\omega,\tau)}{\partial \omega} + \frac{\alpha_w}{\pi}\left(C_A - \frac{\mathbf{T}^2}{2}\right)\tilde{P}(\omega)\tilde{f}^{(T)}(\omega,\tau), \tilde{P}(\omega) \equiv \int_0^1 dz\, z^{\omega} P(z) \tag{4.2}$$

The solution simplifies in the cases $T = 0, 1$ that we shall consider separately.

4.1 Solutions for T=0 and T=1

To start with, for $T = 0$, (4.2) takes a DGLAP form. In the small-x limit, with $P(z) = 1/z \Rightarrow P(\omega) = 1/\omega$, given the initial conditions $\tilde{f}^{(0)}(\omega, \tau = 0) \equiv \tilde{f}_0^{(0)}(\omega)$, we obtain

$$\tilde{f}^{(0)}(\omega,\tau) = \tilde{f}_0^{(0)}(\omega)\, \exp\left[\frac{\alpha_W}{\pi}\frac{C_A}{\omega}\tau\right] \tag{4.3}$$

and, by antitrasforming:

$$x f^{(0)}(x,\tau) = \frac{1}{2\pi i}\int_{c-i\infty}^{c+i\infty} d\omega\, \exp\left[\omega\, \log\frac{1}{x} + \frac{\alpha_W}{\pi}\frac{C_A}{\omega}\,\tau\right]\, \tilde{f}_0^{(0)}(\omega), \quad (T=0) \tag{4.4}$$

When $\log\frac{1}{x} \gg 1$ this integral can be evaluated by a saddle point method and we get the usual double-log DGLAP behaviour, corresponding to a cross-section increase in the small-x region:

$$x f^{(0)}(x,\tau) \simeq \left(\frac{\alpha_w C_A \tau/\pi \log\frac{1}{x}}{4\pi\sqrt{\alpha_w C_A \tau \log\frac{1}{x}}}\right)^{1/2} \exp\left[2\sqrt{\frac{\alpha_w}{\pi} C_A \tau \log\frac{1}{x}}\right] \tilde{f}_0^{(0)}\left(\omega = \sqrt{\frac{\alpha_w C_A \tau}{\pi \log\frac{1}{x}}}\right), \quad (T=0) \tag{4.5}$$

The case $T = 1$ is interesting too, but shows a quite different behaviour whose general form will be found below. Here we note a special small-x solution of eq. (3.5) in the case $P(z) \simeq 1/z$, provided simply by

$$x f^{(1)}(x,\tau) \;=\; F_1 = const. \quad (T=1) \tag{4.6}$$

The reason for such a constant solution is that the flavour factors for virtual and real emission terms become equal for $T = 1$, namely $\boldsymbol{T}^2/2 = C_A - \boldsymbol{T}^2/2 = 1$. Therefore, the $\log 1/x$ evolution factor cancels out between virtual and real emission contributions. A subleading x, τ dependence survives, and is found below for given boundary conditions. However the fact remains that, for $T = 1$, there are no double-log corrections ($\tau\; log\frac{1}{x}$) to the basic form factor factorized in (3.3).

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4.2 Solutions for generic T values

For generic values of T, we can integrate eq. (4.2) to get the general solution

$$\tilde{f}^{(T)}(\omega,\tau)=\Phi\left(\frac{\alpha_w \mathbf{T}^2}{2\pi}\,\tau+\omega\right)\ \exp\left[-\left(\frac{2C_A}{\mathbf{T}^2}-1\right)\ \int_c^{\omega} d\omega'\ \tilde{P}(\omega')\right] \tag{4.7}$$

c being an arbitrary constant and Φ an arbitrary function, to be determined through initial conditions. If we demand that, at $\tau=0$, $\tilde{f}^{(T)}(\omega,\tau=0)=\tilde{f}_0^{(T)}(\omega)$ then, since $C_A=2$:

$$\tilde{f}^{(T)}(\omega,\tau)=\ \tilde{f}_0^{(T)}\left(\omega+\frac{\alpha_w \mathbf{T}^2}{2\pi}\tau\right)\ \ \exp\left[\left(\frac{4}{\mathbf{T}^2}-1\right)\ \int_{\omega}^{\omega+\frac{\alpha_w \mathbf{T}^2}{2\pi}\tau} d\omega' \tilde{P}(\omega')\right] \tag{4.8}$$

and, by antitrasforming to x space, we find that an extra form factor is factorized out as follows:

$$\begin{aligned} xf^{(T)}(x,\tau)\ &=\int_{c-i\infty}^{c+i\infty}\frac{d\omega}{2\pi i}x^{-\omega}\tilde{f}_0^{(T)}\left(\omega+\frac{\alpha_w \mathbf{T}^2}{2\pi}\tau\right)\exp\left[\left(\frac{4}{\mathbf{T}^2}-1\right)\int_{\omega}^{\omega+\frac{\alpha_w \mathbf{T}^2}{2\pi}\tau}d\omega'\tilde{P}(\omega')\right] && (4.9)\\ &=e^{-\frac{\alpha_w \mathbf{T}^2}{2\pi}\tau\log\frac{1}{x}}\int_{c-i\infty}^{c+i\infty}\frac{d\omega}{2\pi i}\ x^{-\omega}\ \tilde{f}_0^{(T)}(\omega)\exp\left[\left(\frac{4}{\mathbf{T}^2}-1\right)\int_{\omega-\frac{\alpha_w \mathbf{T}^2}{2\pi}\tau}^{\omega}d\omega'\tilde{P}(\omega')\right] && (4.10)\end{aligned}$$

If we focus on the most singular part as x becomes small, then $P(z)=\frac{1}{z}$ and $\tilde{P}(\omega)=\frac{1}{\omega}$ so that

$$xf^{(T)}(x,\tau)=e^{-\frac{\alpha_w \mathbf{T}^2}{2\pi}\tau\log\frac{1}{x}}\ \int_{c-i\infty}^{c+i\infty}\frac{d\omega}{2\pi i}\ x^{-\omega}\ \tilde{f}_0^{(T)}(\omega)\ \left(\frac{\omega}{\omega-\frac{\alpha_w \mathbf{T}^2}{2\pi}\tau}\right)^{(\frac{4}{\mathbf{T}^2}-1)} \tag{4.11}$$

Let us first recover the case $T=1$, i.e. $\mathbf{T}^2=2$. Then the solution is:

$$xf^{(1)}(x,\tau)=e^{-\frac{\alpha_w}{\pi}\tau\log\frac{1}{x}}\ \int_{c-i\infty}^{c+i\infty}\frac{d\omega}{2\pi i}\ x^{-\omega}\ \tilde{f}_0^{(1)}(\omega)\ \left(\frac{\omega}{\omega-\frac{\alpha_w}{\pi}\tau}\right),\qquad (T=1) \tag{4.12}$$

Note the pole at $\omega=\alpha_w\tau/\pi$ which, in the small-x region, implies the *cancellation* of the form factor in front for any initial condition $\tilde{f}_0^{(1)}(\omega)$. In particular, if we consider a flat distribution at $\tau=0$, $xf_0^{(1)}(x)=F_1\Rightarrow\tilde{f}_0^{(1)}(\omega)=\frac{F_1}{\omega}$, then $xf^{(1)}(x,\tau)=F_1=const$ also, as in (4.6), so that no large terms proportional to $\log x$ are generated at all.

The case $T=2$ is more involved, but is still manageable analytically for $\tilde{f}_0^{(2)}(\omega)=F_2/\omega$, corresponding to a flat initial distribution. In such case the ω-integral in (4.11) can be expressed in terms of a confluent hypergeometric function as follows

$$\begin{aligned} xf^{(2)}(x,\tau)&=e^{-\frac{3\alpha_w}{\pi}\tau\log\frac{1}{x}}\ \int_{c-i\infty}^{c+i\infty}\frac{d\omega}{2\pi i}\ x^{-\omega}\ \frac{F_2}{\omega}\ \left(\frac{\omega-\frac{\alpha_w}{\pi}\tau}{\omega}\right)^{\frac{1}{3}} && (4.13)\\ &=F_2 e^{-\frac{3\alpha_w}{\pi}\tau\log\frac{1}{x}}F\left[-\frac{1}{3},1,\frac{3\,\alpha_w}{\pi}\tau\log\frac{1}{x}\right]\simeq-\frac{F_2}{3\Gamma(2/3)}\left(\frac{3\,\alpha_w}{\pi}\,\tau\ \log\frac{1}{x}\right)^{-4/3} && (T=2)\end{aligned}$$

where the last behaviour holds for $\alpha_w\tau\log 1/x\gg 1$. The double-log dependence is now nontrivial (see figure 4): starting from the naive form factor, it changes sign, eventually,

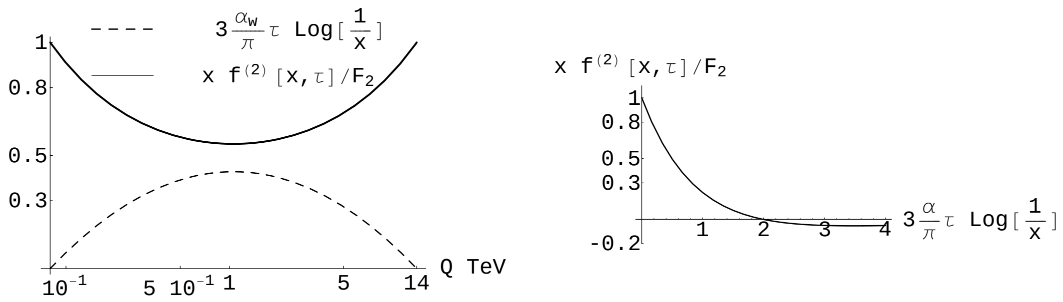

Figure 4: Plots of $\frac{3\alpha_W}{\pi}\tau \log\frac{1}{x}$ and of the structure function $xf^{(2)}(x,\tau)/F_2$ in eq (4.13) as functions of Q for $x = Q/\sqrt{s}$ and $\sqrt{s}$ =14 TeV, and of $xf^{(2)}(x,\tau)/F_2$ in eq (4.13) as function of $\frac{3\alpha_W}{\pi}\tau \log\frac{1}{x}$.

for large (unrealistic) values of $\alpha_W \tau \log 1/x$. The reason for that is that the virtual and real emission flavour factors are of opposite signs.

We have so far considered initial conditions which allow a simple analytical understanding of the solution. In a realistic case, one should take into account the initial luminosities and set up the appropriate initial conditions by projecting out the various T-dependent components of the overlap matrix occurring in eq. (3.3) according to the general formula

$$\mathcal{O}^H = \sum_{t_1,t_2\ldots t_n} O^H_{t_1 t_2 \ldots t_n} \mathcal{P}_{t_1 t_2 \ldots t_n} \tag{4.14}$$

where $\mathcal{O}, \mathcal{P}_{t_1 t_2 \ldots t_n}$ are operators acting on the n external legs indices, and $O_{t_1 t_2 \ldots t_n}$ are the coefficients of the expansion. The projectors satisfy, by definition:

$$\boldsymbol{T}_j^2 \mathcal{P}_{t_1 t_2 \ldots t_n} = t_j(t_j+1)\mathcal{P}_{t_1 t_2 \ldots t_n}, j = 1 \ldots n \qquad \boldsymbol{T}_{\rm tot}\mathcal{P}_{t_1 t_2 \ldots t_n} = 0 \tag{4.15}$$

and have been constructed in various cases in refs [10].

A realistic calculation of the double-log effects by the procedure outlined above is outside the scope of the present paper, and is deferred to further work. We only remark that x-dependent double log effects have been found here in the $T = 2$ component only, but are – in that component — relatively large. Indeed, the size of the effect, relative to the F_2 component introduced in (4.13), can be seen in figure 4. At LHC energies, if we consider x_1 and x_2 to be of the same order, we can approximate $x_1 \sim x_2 \equiv x = Q/(14\,TeV)$; then the variable $\frac{3\,\alpha_w}{\pi}\tau \log 1/x$ has a maximum for $Q^2 = M\sqrt{s} \simeq 1$ TeV and takes the maximum value $\frac{3\,\alpha_w}{8\,\pi}\log^2\frac{s}{M^2} \simeq 0.43$ corresponding to a depletion factor for eq (4.13) of $\sim 0.57\;F_2$. If we include the exponential factor $\exp\left[-3\frac{\alpha_w}{4\pi}\log^2\frac{Q^2}{M^2}\right]$ coming from the factorization of the overlap matrix (3.3), we reach a depletion of $\sim 0.46\;F_2$ that corresponds to corrections at the 50 % level!

Despite such relatively large effects, the magnitude of F_2 itself is subject to severe constraints in an accelerator like LHC, where the "initial" Ws come from quarks in the proton. Since the fermions are isospin doublets, in order to build $T = 2$ we need to identify four fermionic charges: two of them are the initial proton beams, and two must be in the final state. For instance, one could in principle trigger the central hard process at scale Q on forward and backward jets associated to the $q \to W$ transitions in the proton[§] where, in addition, one should measure the jet charges. In such (unrealistic) measurement the $T = 2$ component exists and the extra x-dependent double-logs can be important, as described before. However, if no charge is measured in the final state – as in the flavour-blind experiments considered previously — no $T = 2$ component can occur and no extra double-logs are found, besides those factored out in the Q^2 dependent form factor.

Alternatively, one can look at experiments in which flavour charges are measured in the hard final state, as in W-pair production, possibly associated with a trigger. In such a case, the overlap function has at least six indices, as outlined at the end of Sec 3 and in ref. [6], and a $T = 2$ component is more easily built up. A detailed phenomenological analysis of such kind of processes is yet to be performed.

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5. Small-x evolution in BFKL-type approach

The BFKL approach [12] was originally proposed for massive vector bosons, and has been recently revisited, and applied to electroweak theory and to its symmetry breaking in [13]. Here we work in the $s \gg Q^2 \gg M^2$ regime where global-symmetry restoration is expected and we take a simplified approach, in which all vector bosons have the same mass, which acts as symmetry breaking scale and as infrared cutoff. With such simplifications, and using the notation $Y = log\frac{1}{x}$ and $t = log\frac{\mathbf{k}^2}{M^2}$, the weak isospin BFKL equation can be written in the following form

$$\frac{\partial}{\partial Y}\mathcal{F}^{(T)}(t,Y) = -\frac{\alpha_w}{\pi}\frac{\mathbf{T}^2}{2}\, t\, \mathcal{F}^{(T)}(t,Y) + \frac{\alpha_w}{\pi}\left(C_A - \frac{\mathbf{T}^2}{2}\right)\int \frac{d^2\mathbf{k}'}{\pi}\,\mathbf{K}(\mathbf{k},\mathbf{k}')\,\mathcal{F}^{(T)}(t',Y) \tag{5.1}$$

where $t' = log\frac{\mathbf{k}'^2}{M^2}$. We note that the diagonal term in the r.h.s. is proportional to the vector boson reggeon intercept $\omega_V(\mathbf{k}^2) = -(\alpha_w/\pi)\log \mathbf{k}^2/M^2$, and becomes identical to it for $T = 1, \boldsymbol{T}^2 = 2$. Furthermore, the flavour factors are, once again, the same as in the eikonal and collinear evolution equation (3.5) analyzed before.

Finally, the (regularized) kernel $\mathbf{K}$ has the spectral representation

$$\mathbf{K}(\mathbf{k}',\mathbf{k}) = \frac{1}{\mathbf{k}^2}\int \frac{d\gamma'}{2\pi\, i}\,\chi(\gamma')\,\left(\frac{\mathbf{k}^2}{\mathbf{k}'^2}\right)^{\gamma'} \tag{5.2}$$

where $\chi(\gamma) = \frac{1}{\gamma} + 2\,\psi(1) + \psi(1+\gamma) - \psi(1-\gamma) \sim \frac{1}{\gamma} + 0(\gamma^2)$ (with ψ the digamma function) is the BFKL eigenvalue function , according to the equation

$$\int \frac{d\mathbf{k}'^{\,2}}{\pi}\,\mathbf{k}'^{\,2\,(\gamma-1)}\,\mathbf{K}(\mathbf{k},\mathbf{k}') = \chi(\gamma)\,\mathbf{k}^{2\,(\gamma-1)} \tag{5.3}$$

It is then convenient to introduce the γ-representation ($Y \equiv \log\frac{1}{x}$)

$$k^2\mathcal{F}(k^2,Y) = \int \frac{d\gamma}{2\pi i}e^{\gamma t}\tilde{\mathcal{F}}(\gamma,Y); \qquad \tilde{\mathcal{F}}(\gamma,Y) = \int_0^\infty dk^2 e^{-\gamma t}\mathcal{F}(k^2,Y) \tag{5.4}$$

[§]We wish to thank the referee for a remark related to this point, that helped us in understanding the cases for which the $T = 2$ component is relevant.

and to rewrite eq. (5.1) as a differential equation

$$\frac{\partial}{\partial Y}\tilde{\mathcal{F}}^{(T)}(\gamma,Y) = \frac{\alpha_w}{\pi}\,\frac{\mathbf{T}^2}{2}\,\frac{\partial}{\partial\gamma}\tilde{\mathcal{F}}^{(T)}(\gamma,Y) + \frac{\alpha_w}{\pi}\left(C_A - \frac{\mathbf{T}^2}{2}\right)\,\chi(\gamma)\,\tilde{\mathcal{F}}^{(T)}(\gamma,Y) \tag{5.5}$$

This equation is now of the same form as eq. (4.2), with the variables τ, ω interchanged with Y, γ and, by the same manipulations, admits the general solution

$$\begin{aligned} k^2\mathcal{F}^{(T)}(k^2,Y) \equiv \mathcal{F}^{(T)}(t,Y) = & \int_{c-i\infty}^{c+i\infty}\frac{d\gamma}{2\pi i}\,e^{\gamma t}\,\tilde{\mathcal{F}_0}^{(T)}\left(\gamma + \frac{\alpha_w\mathbf{T}^2}{2\pi}Y\right) \\ & \times\exp\left[\left(\frac{4}{\mathbf{T}^2}-1\right)\int_{\gamma}^{\gamma+\frac{\alpha_w\mathbf{T}^2}{2\pi}Y} d\gamma'\chi(\gamma')\right] \qquad (5.6) \\ = & \; e^{-\frac{\alpha_w\mathbf{T}^2}{2\pi}tY}\int_{c-i\infty}^{c+i\infty}\frac{d\gamma}{2\pi i}\,e^{\gamma t}\,\tilde{\mathcal{F}}_0^{(T)}(\gamma) \\ & \times\exp\left[\left(\frac{4}{\mathbf{T}^2}-1\right)\int_{\gamma-\frac{\alpha_w\mathbf{T}^2}{2\pi}Y}^{\gamma} d\gamma'\chi(\gamma')\right] \qquad (5.7) \end{aligned}$$

Such expressions look very similar to the general solution for the DGLAP-type density $f(x,\tau)$, with the crucial difference that the initial condition is now set at $Y=0$ instead of $\tau=0$. This means that, in order to relate the two kinds of densities one should consistently relate the boundary conditions too. In particular, in the collinear limit for which $\chi(\gamma)\simeq\frac{1}{\gamma}$, we obtain

$$\mathcal{F}^{(T)}(t,Y) = e^{-\frac{\alpha_w}{\pi}\,\frac{\mathbf{T}^2}{2}\,Y\,t}\int\frac{d\gamma}{2\pi\,i}\,\tilde{\mathcal{F}}^{(T)}(Y=0,\gamma)\;e^{\gamma\,t}\left(\frac{\gamma}{\gamma-\frac{\alpha_w}{\pi}\frac{\mathbf{T}^2}{2}\,Y}\right)^{\frac{4}{\mathbf{T}^2}-1} \tag{5.8}$$

which will now be related to the solution (4.11) in the DGLAP approach by a proper choice of initial condition.

5.1 Solutions for $T=0$ and $T=1$

The $T=0$ equation in (5.8) is QCD-like, and reads

$$\mathcal{F}^{(0)}(t,Y) = \frac{1}{2\pi i}\int_{c-i\infty}^{c+i\infty} d\gamma\;\exp\left[\gamma t + \frac{\alpha_W}{\pi}\frac{C_A}{\gamma}Y\right]\;\tilde{\mathcal{F}}_0^{(0)}(\gamma),\quad (T=0) \tag{5.9}$$

to be compared to the DGLAP-type solution (4.4). The initial condition corresponding to $f_0(\omega) = F_0/\omega$ — a constant in Y space — turns out to be simply $\tilde{\mathcal{F}}_0(\gamma) = F_0$ — a delta-function in t space.

The corresponding saddle point estimates are, according to eq. (4.5),

$$\begin{aligned} xf^{(0)}(x,t) &\simeq F_0\;\left(4\pi\sqrt{\frac{\alpha_w}{\pi}C_A tY}\right)^{-1/2}\;\exp\left[2\sqrt{\frac{\alpha_w}{\pi}C_A tY}\right] \qquad (5.10) \\ k^2\mathcal{F}^{(0)}(k^2,Y) &\simeq \left(\frac{\alpha_w C_A Y}{\pi t}\right)^{1/2} xf^{(0)}(x,t)\;\simeq\;\frac{\partial xf^{(0)}(x,t)}{\partial t}\qquad (T=0) \end{aligned}$$

thus justifying the customary name of "unintegrated PDF" for $\mathcal{F}(t, Y)$ in this case.

The $T = 1$ case is again simplified by the presence of the simple pole at $\gamma = \alpha_w Y/\pi$ in (5.8). By taking the initial condition $\tilde{\mathcal{F}}_0^{(1)}(\gamma) = F_1$, corresponding to a delta function in t-space, we get the solution

$$k^2 \mathcal{F}^{(1)}(k^2, Y) = F_1(\delta(t) + \frac{\alpha_w}{\pi} Y \theta(t)) \qquad (T = 1) \tag{5.11}$$

which can be easily double-checked by using the collinear approximation $K \simeq \Theta(k^2 - k'^2)/k^2$ for the BFKL kernel.

Let us remark that the solution (5.11) is not the only one without double-logs. From eq. (5.6) we obtain a particular Y-independent solution

$$\mathcal{F}^{(1)}(t, Y) = \mathcal{F}_0^{(1)}(t) = \int_{c-i\infty}^{c+i\infty} \frac{d\gamma}{2\pi i} e^{\gamma t} \exp[-\int^{\gamma} d\gamma' \chi(\gamma')] \tag{5.12}$$

which, in the collinear approximation $\chi(\gamma) \simeq 1/\gamma$, yields just $\mathcal{F}^{(1)}(t, Y)$ =const. This kind of solution corresponds, in our simplified approach, to the gauge-boson Regge pole (having unit intercept) and realizes, therefore, the so-called bootstrap of the adjoint representation [12, 17, 13].

5.2 Solutions for generic T values

The previous examples suggest to look for a general relation between BFKL-type and DGLAP-type densities by assuming the related initial conditions

$$\begin{aligned} x f^{(T)}(x, \tau = 0) &= F_T, \qquad f_0^{(T)}(\omega) = F_T/\omega, \\ \mathcal{F}^{(T)}(t, Y = 0) &= F_T\ \delta(t), \quad \tilde{\mathcal{F}}_0^{(T)}(\gamma) = F_T \end{aligned} \tag{5.13}$$

In fact, we notice that the expressions in eq. (4.11) (eq. (5.8)) can be almost identified by the rescaling $\omega \to \alpha_w \boldsymbol{T}^2 \tau/\pi\ \eta$ ($\gamma \to \alpha_w \boldsymbol{T}^2 Y/\pi\ \eta$), which singles out the double-log variable $\alpha_w \boldsymbol{T}^2 \tau Y/\pi$. The two kinds of densities are thus simply related, except for a γ-integral Jacobian factor which is compensated by a t-derivative in the ω-integral as follows:

$$e^{\frac{\alpha}{\pi}\frac{\mathbf{T}^2}{2} Y\, t}\ \mathcal{F}^{(T)}(t, Y) = \frac{\partial}{\partial t}\left(e^{\frac{\alpha}{\pi}\frac{\mathbf{T}^2}{2} Y\, t}\ \ x f^{(T)}(x, t)\right) \tag{5.14}$$

This equation extends to generic T values the identification of $\mathcal{F}^{(T)}(t, Y)$ as a sort of "unintegrated" density, compared to the integrated distribution function $x f^{(T)}(x, t)$. Therefore, with a proper correspondence of initial conditions, full consistency of τ-evolution with Y-evolution is achieved.

6. Conclusions

We have investigated the structure of enhanced EW corrections to a basic Drell-Yan-type inclusive process (like $WW(s) \to \text{jet}(Q) + X$, with proper warnings about initial state luminosities at LHC) in the kinematical limit where $x^2 \equiv x_1 x_2 = Q^2/s \ll 1$. This regime is characterized by three different scales $s \gg Q^2 \gg M^2$ and the gauge boson emission generates several large logarithms, of high-energy type $\sim \log \frac{s}{Q^2} \sim \log(x_1 x_2)$ and of infrared or collinear type $\sim t \equiv \log \frac{Q^2}{M^2}$. Due to its nonabelian nature, the eikonal W-emission (section 3) naively predicts the presence of various kinds of uncanceled double log corrections, $\log^2 x_i$, $\log x_i \, \log \frac{Q^2}{M^2}$ and $\log^2 \frac{Q^2}{M^2}$, arising in the eikonal exponent, of type $\log^2 \frac{s}{M^2}$. We have first justified a factorized structure of the cross-section, in which the double-log form factor occurs at scale Q^2, while the "incoming parton" distribution functions (which also involve leptons and gauge bosons) only have collinear and high-energy logs.

Then, by solving both the EW collinear evolution equations [10] and the EW BFKL dynamics [13] (sections 4 and 5), we have explicitly computed the dependence of the PDFs on such enhanced variables, and we find peculiar features, depending on the values of the total t-channel weak isospin $\mathbf{T}^2 = T(T+1)$:

- For $T = 0$ the EW corrections have the same structure as the QCD ones (4.5) and thus show a customary double-log enhancement of mixed type ($\sim \alpha_w \log \frac{Q^2}{M^2} \log \frac{1}{x}$) .
- For $T = 1$, potential large and negative $\alpha_w t \log \frac{1}{x}$ corrections can appear but, due to the fact that the Casimir charges for real and virtual W-emission are equal, a cancellation mechanism is at work (4.12), leaving only the exponential form factor ($\exp\left[-\frac{\alpha_w}{2\pi} \log^2 \frac{Q^2}{M^2}\right]$) already incorporated in the factorization formula for the overlap matrix, eq (3.3).
- For $T = 2$ the Casimir charges for real and virtual W-emission are different (and of opposite sign) so the previous mechanism of cancellation fails and large $\alpha_w t \log \frac{1}{x}$ and non-trivial corrections to the form factor at scale Q^2 can be present. The relative magnitude of such effects is pretty large at LHC energies, as shown in figure 4.

The above analysis tells us that, with relatively low Q^2, important x-dependent EW corrections can be present only for cross sections initiated by two transverse gauge bosons, being the only partons supporting the $T = 2$ total t-channel isospin component. For instance, in ref [5] we have already investigated the isospin decomposition of the partonic cross section $WW \to f\bar{f}$, inclusive over the final fermions, at the double log level.

On the other hand, at the LHC accelerator, the "initial" Ws come from quarks in the proton, and this provides some averaging of the initial W states which causes the $T = 2$ component to *vanish* altogether (see section 4), unless the charges of the decay products of the $q \to W$ transitions are measured. We conclude, therefore, that the extra double-logs cancel out, eventually, in flavour-blind experiments at LHC, which are only sensitive to the Q^2-dependent EW form factor.

Instead, the $T = 2$ component and the extra-double logs are turned on if the decay charges mentioned before are measured, or if proper flavour charges are identified in the hard final state. In such cases the overlap function has more than two charged t-channels, possibly with $T = 2$, which can be in the initial and in the final state, and detailed phenomenological applications are yet to be studied.

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References

[1] M. Kuroda, G. Moultaka and D. Schildknecht, *Direct one loop renormalization of* $\mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$ *four fermion processes and running coupling constants*, *Nucl. Phys.* **B 350** (1991) 25;
G. Degrassi and A. Sirlin, *Gauge invariant selfenergies and vertex parts of the Standard Model in the pinch technique framework*, *Phys. Rev.* **D 46** (1992) 3104;
A. Denner, S. Dittmaier and R. Schuster, *Radiative corrections to* $\gamma\gamma \to W^+W^-$ *in the electroweak standard model*, *Nucl. Phys.* **B 452** (1995) 80 [hep-ph/9503442];
A. Denner, S. Dittmaier and T. Hahn, *Radiative corrections to* $Z\,Z \to Z\,Z$ *in the electroweak standard model*, *Phys. Rev.* **D 56** (1997) 117 [hep-ph/9612390];
A. Denner and T. Hahn, *Radiative corrections to* $W^+W^- \to W^+W^-$ *in the electroweak standard model*, *Nucl. Phys.* **B 525** (1998) 27 [hep-ph/9711302];
W. Beenakker, A. Denner, S. Dittmaier, R. Mertig and T. Sack, *High-energy approximation for on-shell W pair production*, *Nucl. Phys.* **B 410** (1993) 245;
W. Beenakker, A. Denner, S. Dittmaier and R. Mertig, *On shell W pair production in the TeV range*, *Phys. Lett.* **B 317** (1993) 622;
M. Beccaria, G. Montagna, F. Piccinini, F.M. Renard and C. Verzegnassi, *Rising bosonic electroweak virtual effects at high energy* e^+e^- *colliders*, *Phys. Rev.* **D 58** (1998) 093014 [hep-ph/9805250].

[2] P. Ciafaloni and D. Comelli, *Sudakov enhancement of electroweak corrections*, *Phys. Lett.* **B 446** (1999) 278 [hep-ph/9809321].

[3] V.S. Fadin, L.N. Lipatov, A.D. Martin and M. Melles, *Resummation of double logarithms in electroweak high energy processes*, *Phys. Rev.* **D 61** (2000) 094002 [hep-ph/9910338];
P. Ciafaloni and D. Comelli, *Electroweak Sudakov form factors and nonfactorizable soft QED effects at NLC energies*, *Phys. Lett.* **B 476** (2000) 49 [hep-ph/9910278];
J.H. Kuhn, A.A. Penin and V.A. Smirnov, *Summing up subleading Sudakov logarithms*, *Eur. Phys. J.* **C 17** (2000) 97 [hep-ph/9912503];
J.H. Kuhn, S. Moch, A.A. Penin and V.A. Smirnov, *Next-to-next-to-leading logarithms in four-fermion electroweak processes at high energy*, *Nucl. Phys.* **B 616** (2001) 286 [*Erratum ibid.* **B 648** (2003) 455] [hep-ph/0106298];
M. Melles, *Electroweak radiative corrections in high energy processes*, *Phys. Rept.* **375** (2003) 219 [hep-ph/0104232];
J.-y. Chiu, F. Golf, R. Kelley and A.V. Manohar, *Electroweak corrections in high energy processes using effective field theory*, *Phys. Rev.* **D 77** (2008) 053004 [arXiv:0712.0396].

[4] M. Ciafaloni, P. Ciafaloni and D. Comelli, *Bloch-Nordsieck violating electroweak corrections to inclusive TeV scale hard processes*, *Phys. Rev. Lett.* **84** (2000) 4810 [hep-ph/0001142]; *Electroweak double logarithms in inclusive observables for a generic initial state*, *Phys. Lett.* **B 501** (2001) 216 [hep-ph/0007096]; *Electroweak Bloch-Nordsieck violation at the TeV scale: 'Strong' weak interactions?*, *Nucl. Phys.* **B 589** (2000) 359 [hep-ph/0004071]; *Enhanced*

electroweak corrections to inclusive boson fusion processes at the TeV scale, *Nucl. Phys.* **B 613** (2001) 382 [hep-ph/0103316]; *Bloch-Nordsieck violation in spontaneously broken Abelian theories*, *Phys. Rev. Lett.* **87** (2001) 211802 [hep-ph/0103315];
P. Ciafaloni, D. Comelli and A. Vergine, *Sudakov electroweak effects in transversely polarized beams*, *JHEP* **07** (2004) 039 [hep-ph/0311260];
M. Ciafaloni, *Infrared sensitive physics in QCD and in electroweak theory*, *Lect. Notes Phys.* **737** (2008) 151 [hep-ph/0612067].

[5] M. Ciafaloni, P. Ciafaloni and D. Comelli, *Electroweak double logarithms in inclusive observables for a generic initial state*, *Phys. Lett.* **B 501** (2001) 216 [hep-ph/0007096].

[6] P. Ciafaloni and D. Comelli, *The importance of weak bosons emission at LHC*, *JHEP* **09** (2006) 055 [hep-ph/0604070].

[7] U. Baur, *Weak boson emission in hadron collider processes*, *Phys. Rev.* **D 75** (2007) 013005 [hep-ph/0611241];
R.S. Thorne, *The role of uncertainties in parton distribution functions*, arXiv:0711.2986.

[8] S. Pozzorini, *Electroweak radiative corrections at high energies*, hep-ph/0201077;

[9] A. Denner, B. Jantzen and S. Pozzorini, *Two-loop electroweak Sudakov logarithms for massive fermion scattering*, arXiv:0801.2647.

[10] M. Ciafaloni, P. Ciafaloni and D. Comelli, *Towards collinear evolution equations in electroweak theory*, *Phys. Rev. Lett.* **88** (2002) 102001 [hep-ph/0111109];
P. Ciafaloni and D. Comelli, *Electroweak evolution equations*, *JHEP* **11** (2005) 022 [hep-ph/0505047].

[11] V. Gribov and L.N. Lipatov, *Deep inelastic e p scattering in perturbation theory*, *Sov. J. Nucl. Phys.* **15** (1972) 438 [*Yad. Fiz.* **15** (1972) 781];
L.N. Lipatov, *The parton model and perturbation theory*, *Sov. J. Nucl. Phys.* **20** (1975) 94 [*Yad. Fiz.* **20** (1974) 181];
G. Altarelli and G. Parisi, *Asymptotic freedom in parton language*, *Nucl. Phys.* **B 126** (1977) 298;
Y.L. Dokshitzer, *Calculation of the structure functions for deep inelastic scattering and e^+e^- annihilation by perturbation theory in quantum chromodynamics* (in Russian), *Sov. Phys. JETP* **46** (1977) 641 [*Zh. Eksp. Teor. Fiz.* **73** (1977) 1216].

[12] L.N. Lipatov, *Reggeization of the vector meson and the vacuum singularity in nonabelian gauge theories*, *Sov. J. Nucl. Phys.* **23** (1976) 338 [*Yad. Fiz.* **23** (1976) 642];
E.A. Kuraev, L.N. Lipatov and V.S. Fadin, *The Pomeranchuk singularity in nonabelian gauge theories*, *Sov. Phys. JETP* **45** (1977) 199 [*Zh. Eksp. Teor. Fiz.* **72** (1977) 377];
I.I. Balitsky and L.N. Lipatov, *The Pomeranchuk singularity in quantum chromodynamics*, *Sov. J. Nucl. Phys.* **28** (1978) 822 [*Yad. Fiz.* **28** (1978) 1597].

[13] J. Bartels, L.N. Lipatov and K. Peters, *The Pomeranchuk singularity and vector boson reggeization in electroweak theory*, *Nucl. Phys.* **B 772** (2007) 103 [hep-ph/0610303].

[14] A. Bassetto, M. Ciafaloni and G. Marchesini, *Jet structure and infrared sensitive quantities in perturbative QCD*, *Phys. Rept.* **100** (1983) 201.

[15] S. Catani and M. Ciafaloni, *Generalized coherent state for soft gluon emission*, *Nucl. Phys.* **B 249** (1985) 301;
G. Sterman, *Summation of large corrections to short distance hadronic cross-sections*, *Nucl. Phys.* **B 281** (1987) 310;

J. Botts and G. Sterman, *Hard elastic scattering in QCD: leading behavior*, *Nucl. Phys.* **B 325** (1989) 62;
R. Bonciani, S. Catani, M.L. Mangano and P. Nason, *Sudakov resummation of multiparton QCD cross sections*, *Phys. Lett.* **B 575** (2003) 268 [hep-ph/0307035].

[16] Y.L. Dokshitzer and G. Marchesini, *Soft gluons at large angles in hadron collisions*, *JHEP* **01** (2006) 007 [hep-ph/0509078].

[17] V.S. Fadin and R. Fiore, *The generalized nonforward BFKL equation and the *bootstrap* condition for the gluon Reggeization in the NLLA*, *Phys. Lett.* **B 440** (1998) 359 [hep-ph/9807472].